Stability Analysis of Single-Phase Thermosyphon Loops by Finite Difference Numerical Methods

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ABSTRACT

In this paper, examples of the application of finite difference numerical methods in the analysis of stability of single-phase natural circulation loops are reported. The problem is here addressed for its relevance for thermal-hydraulic system code applications, in the aim to point out the effects of truncation error on stability prediction. The methodology adopted for analyzing in a systematic way the effect of various finite difference discretization can be considered the numerical analogue of the usual techniques adopted for PDE stability analysis. Three different single-phase loop configurations are considered involving various kinds of boundary conditions. In one of these cases, an original dimensionless form of the governing equations is proposed, adopting the Reynolds number as a flow variable. This allows for an appropriate consideration of transition between laminar and turbulent regimes, which is not possible with other dimensionless forms, thus enlarging the field of validity of model assumptions.

1. INTRODUCTION

Single-phase thermosyphon loops are an interesting example of systems in which fluid dynamic instabilities may arise as a consequence of delayed feedback mechanisms. An explanation of the physical reasons for instability was given by Pierre Welander [1], who coped with the problem of a limiting case of rectangular thermosyphon loop with point heated and cooled sections (source and sink).

According to his explanation, instability in symmetrically heated single-phase loops around steady-state conditions can be ascribed to the formation of perturbed temperature pockets at the exit of the source or the sink, which travel at a different velocity than the one imposed by flow rate. Then, their amplitude is progressively amplified or damped, according to the specific geometrical and physical characteristics of the system, during the subsequent heating and cooling cycles. The observed instability is then characterized by chaotic behaviour involving large flow and temperature oscillations, together with flow reversals.

Natural circulation in single-phase conditions has a great interest in heat transfer applications, including solar energy conversion and passive nuclear reactor cooling. In the latter case, the choice to rely only on passive mechanisms to operate a reactor or managing an accident, poses the problem to demonstrate that in the addressed conditions the involved flow regimes will be enough regular and stable to accomplish with their intended safety function. Therefore, in these systems the possible occurrence of instabilities with a chaotic and, consequently, basically unpredictable nature must be ruled out by design.

In addition, single-phase natural circulation instability phenomena have characteristics similar to the features of other fluid dynamic instabilities of relevance in engineering (e.g., boiling channel instabilities). In these cases, the capability to predict the conditions for stable operation is a critical issue for design and may be pursued by the adoption of both frequency-domain and time-domain techniques.

Frequency-domain techniques are capable to establish the threshold values of physical parameters leading to neutral stability. This important information is obtained by a standard procedure, involving linearization of governing partial differential equations (PDEs) by perturbation, integration and solution of a complex valued characteristic equation. Only few hints on the actual transient behaviour of unstable systems may be obtained by this technique (e.g., frequency of the fastest growing perturbation, degree of amplification of small perturbations), since non-linear behaviour beyond the stability threshold is completely disregarded.

On the other hand, time-domain techniques involve the transient solution of the non-linear equations describing the dynamic behaviour of the system. In this case, the description of the time evolution can be obtained through the numerical discretization of balance equations, very often performed making use of finite difference methods. In many
cases, the system to be described is too complex and the necessary description too detailed to make feasible other techniques for predicting transient behaviour. However, the price to achieve this goal by space and time numerical discretization is the introduction of truncation errors, which may considerably change the predicted dynamics.

In this paper, sample applications of finite difference numerical methods for evaluating the linear stability conditions in single-phase thermosyphon loops are presented. The method of analysis, discussed in a forthcoming paper [2], is based on the linearization by perturbation around a selected fixed point of the finite difference equations obtained discretizing the PDEs governing the transient behaviour of the addressed systems. In this respect, this method can be considered the numerical analogue of the usual linear stability analysis of PDE problems and has also similarity with the techniques adopted for discussing stability of numerical methods.

The examples of application reported in the paper are related to three different natural circulation loops (Figure 1) whose characteristics have been thoroughly analyzed theoretically and, in some cases, also experimentally by other investigators [3].

The first considered loop (Figure 1a) is the one studied in the '60s by Welander in the above mentioned paper [1]. It is a very schematic representation of a thermosyphon loop, consisting in point heat source and sink with imposed wall temperature and heat transfer coefficient. Source and sink are connected by two adiabatic legs along which the fluid is transported by pure advection due to the resulting buoyancy forces. This loop has been considered by the Authors in previous works [4] [2] as a benchmark problem for studying the effect of truncation error on stability prediction. The main results from this work will be shortly summarized to introduce the discussion on new developments.

The second considered system (Figure 1b) consists in a rectangular loop with constant heat flux heating at the bottom and heat exchanger cooling at the top. This kind of arrangement, including mixed boundary conditions, is the more realistic one from the point of view of practical feasibility and has been the subject of very interesting theoretical and experimental studies ([5], [6], [7]).

Finally, the third considered loop (Figure 1c) is perhaps the most popular example of natural circulation single phase loop and consists in a toroidal loop with asymmetric heating [8] [9]. The version of the problem addressed in this paper is the one with imposed heat flux in both the heated and the cooled lengths [10] and has been chosen to fully cover the panorama of the possible boundary conditions.

2. EQUATIONS OF LOOP DYNAMICS

2.1 Welander's Problem

The dimensionless equations representing the dynamics of the loop in Figure 1a are the following:

\[
\frac{dq}{d\tau} = \alpha \int_{0}^{1} \theta \, ds - \varepsilon \, q^{\xi} \quad (\tau > 0) \quad (1)
\]

for momentum and

\[
\frac{\partial \theta}{\partial \tau} + q \frac{\partial \theta}{\partial s} = 0 \quad (0 < s < 1, \tau > 0) \quad (2)
\]

for energy, where the following definitions apply

\[
s = \frac{S}{L} \quad \tau = \frac{t}{L^2} \quad q = \frac{Q}{\kappa \, A \, \Delta S} \quad \theta = \frac{T - T_0}{\Delta T} \quad (3)
\]

\[
\alpha = \frac{g \, \beta \, \Delta T \, L}{2 \, (\kappa \, \Delta S)^2} \quad \varepsilon = \frac{\rho \, D \, \kappa \, \Delta S}{\mu} \left(1 - b\right) \quad \varepsilon_{\text{Wel}} \quad \xi = 2 - b \quad (6)
\]

Here, \(\varepsilon_{\text{Wel}}\) is the friction dimensionless parameter defined by Welander as

![Figure 1 - Considered natural circulation single phase loops](image)

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The Boussinesq approximation for fluid density variation is accepted. Integration of the PDE momentum equation along the loop and the adoption of a macroscopic friction law lead to equation (1), making the flow rate dependent only on time, while \( \omega \) is a function of both \( s \) and \( \tau \). The dimensionless variables \( \alpha \) and \( \varepsilon \) are identified with a buoyancy and a friction parameter, which fully characterize from the physical point of view the system, and \( \zeta \) is a friction law exponent.

The developments needed to reach the above equations are very similar to the ones suggested by Welander in his paper (see [2]). With respect to the original Welander's treatment, relating only to laminar flow, the above equations have been derived considering a general friction law having the form:

\[
f' = \frac{a}{b} \frac{\tau}{\text{Re}}
\]

which results in the following definition for \( \zeta \):

\[
\zeta = 2 - b
\]

In this work, the value adopted for \( \zeta \) is 1.75, coherently with the adoption of the Blasius law for wall friction.

An interesting feature of this problem is that the temperature distribution along the loop is antisymmetric with respect to the center of the loop. In particular, Welander suggested (an it is possible to rigorously show) that if the temperature distribution along the loop is antisymmetric at the beginning of the transient it will keep antisymmetric during the whole time evolution. This allows for solving the energy balance equation along the ascending leg only, i.e. for \( s \in (0,1) \).

Considering antisymmetry, the boundary conditions for temperature are imposed only at the inlet and at the outlet of the ascending leg, that is in the points just after the source (\( s=0^+ \)) and before the sink (\( s=1^- \)). Following the treatment by Welander, these boundary conditions have the form:

\[
\theta(0^+,\tau) + \theta(1^-;\tau) = (1 + \theta(1^-;\tau)) \left( \frac{1}{1 - e^{\beta \tau}} \right) \quad \text{for } q \geq 0, \tau > 0
\]

\[
\theta(0^-;\tau) + \theta(1^-;\tau) = (-1 + \theta(0^+;\tau)) \left( \frac{1}{1 - e^{\beta \tau}} \right) \quad \text{for } q < 0, \tau > 0
\]

The above relationships specify the effects of the source and the sink on the flowing fluid and have been derived by Welander on the basis of a quasi-steady approach. They are justified by assuming that the source and the sink have an infinitesimal length, even though their heat transfer capability is still assumed to be finite.

Steady-states involve positive, negative and zero flow rate conditions. In particular, the negative flow fixed point is just the mirror image of the positive flow one. Therefore, neglecting the zero flow case which is not relevant for heat transfer applications, the greatest interest is in establishing the relationships for stability of the positive flow steady-state conditions.

2.2 Rectangular Loop with Mixed Boundary Conditions

The dimensionless balance equations for this loop have been developed in a previous work [6], in which the scaling laws to be assumed to reduce experimental data concerning single-phase natural circulation loops have been established.

Different dimensionless parameters were identified as the relevant ones in this purpose; these include geometrical aspect ratios of the loop and of the heating and of the cooling sections, the length over diameter ratio and modified Grashof and Stanton numbers. The latter quantities are found the key parameters which determine stability.

The main assumptions at the basis of the development of balance equations are the usual ones in this field:

- the legs are assumed to be adiabatic;
- the Boussinesq approximation is applied;
- axial heat conduction in the fluid is neglected;
- the flow is one-dimensional.

With the above assumptions the momentum and energy balance equations assume the form:

\[
\begin{align*}
\frac{d\omega}{d\tau} & = \frac{\text{Gr}_m}{\text{Re}_m^3} \int_{\Delta T} \frac{\partial \theta}{\partial s} - \frac{p}{\text{Re}_m^b} \frac{L}{D} \Delta \omega^{2-b} \frac{b}{2} \\
\frac{\partial \theta}{\partial s} + \phi \omega \frac{\partial \theta}{\partial s} & = \frac{V}{V_{\text{heater}}} (\text{heater}) \\
\frac{\partial \theta}{\partial s} + \phi \omega \frac{\partial \theta}{\partial s} & = -St_m \theta (\text{cooler}) \\
\frac{\partial \theta}{\partial s} + \phi \omega \frac{\partial \theta}{\partial s} & = 0 (\text{legs})
\end{align*}
\]

where

\[
\omega = \frac{W}{W_{ss}}, \quad z = \frac{Z}{H}, \quad s = \frac{S}{H}
\]

\[
\tau = \frac{W_{ss}}{V \rho}, \quad \theta = \frac{T - T_s}{(\Delta T_{\text{heater}})^{ss}}, \quad \text{Re}_{ss} = \frac{D W_{ss}}{A \mu}
\]

\[
\text{Gr}_m = \frac{D^2 \rho^2 g b P H}{\mu^3 A C_p}, \quad \text{St}_m = \frac{4 N_u_m}{\text{Re}_{ss} \text{Pr}}
\]

\[
N_u_m = \frac{u_i L}{k}, \quad \text{Pr} = \frac{C_p \mu}{k}, \quad \phi = \frac{V}{A H}
\]

In deriving the above equations, a friction law having the form

\[
f' = \frac{p}{\text{Re}^b}
\]

was assumed.

Steady-state conditions involve both positive and negative flow rates and, due to the symmetry of heating with respect to the vertical axis, the two fixed points are the mirror image of each other. It is easily understood that the zero flow conditions cannot be steady in the present case owing to the presence of the imposed heat flux heating.

2.3 Toroidal Loop with Imposed Heat Flux

The present problem has been studied both theoretically and experimentally by several Authors considering different kinds of boundary conditions [8], [9], [10], [11]. In the work by Sen et al. (1985) [10], the specific boundary conditions addressed in this paper are considered.

With respect to the dimensionless form of governing equations adopted in that paper, the need was felt here to establish a set of dimensionless groups more directly related...
to engineering applications. In fact, many of the sets of
dimensionless groups previously proposed for single-phase
natural circulation loops have the following disadvantages:
• it is not very intuitive to locate in the dimensionless
parameter space the region of interest for practical
applications;
• it may be difficult, or even impossible, to properly
account for physical phenomena which are known to be
governed by parameters other than those adopted in the
dimensionless equations.
The former of the two above observations becomes clear
while trying to establish whether interesting phenomena
devised in theoretical analyses may be possibly confirmed
by real life experiments. The latter observation points out
that phenomena as flow and heat transfer transition regime
are hardly accounted for in analytical models, unless the
appropriate dimensionless groups governing the transition
are included.

A typical case is related to the choice of the wall friction
law. In the available literature dealing with stability of
natural circulation in single-phase loops, a choice is always
made about the form of the wall friction law to be used in
analytical developments and also about the coefficients to be
adopted; examples are reported above for the two other
considered natural circulation loops. This limits the analysis
to either laminar flow or turbulent flow conditions, without
any possibility to account for transition between them, since
the Reynolds number is not used as an independent flow
variable.

Trying to compensate for the above discussed
deficiencies in available models, a step forth towards a
greater realism in the analytical description of natural
circulation loops is tried in the present paper, selecting the
Reynolds number as main flow variable in the dimensionless
definitions. This limits the analysis
to either laminar flow or turbulent flow conditions, without
any possibility to account for transition between them, since
the Reynolds number is not used as an independent flow
variable.

Concerning the boundary conditions it is assumed that
the heat flux is uniform and constant in both the heating and
the cooling sections. With these conditions, the problem has
two steady-state solutions, one for positive and the other for
negative flow, in a limited range of γ, which is the angle of asymmetry in heating.

As well known, the problem is amenable to a very simple
modal solution, in which only three equations are found to
fully govern the dynamics of the loop [8]. With the present
notion, assuming the following expression dimensionless
heat flux (corresponding to uniform heating and cooling in
the two halves of the loop)

\[ q^*(s) = \sum_{l=0}^{\infty} \frac{4 (-1)^k}{(2l+1) \pi} \cos \left[ (2l+1) \pi \left( s - \frac{\gamma}{\pi} \right) \right] \]  (26)

the three modal ODEs are obtained by expanding the
temperature distribution in series of trigonometric functions
and applying the usual weighting technique

\[ \frac{dR}{d\tau} = Gr \frac{\Psi_1}{2} \frac{1}{D} f'(Re) Re |Re| \]
\[ \frac{d\Psi_1}{d\tau} = \pi Re \Omega_1 - \pi^2 Fo \Psi_1 + \frac{4}{\pi} \sin \gamma \]  (27)
\[ \frac{d\Omega_1}{d\tau} = - \pi Re \Psi_1 - \pi^2 Fo \Omega_1 + \frac{4}{\pi} \cos \gamma \]

were \( \Psi_1 \) and \( \Omega_1 \) are the coefficients of the sinus and cosinus
of the first mode. In particular, the case with \( Fo = 0 \) is
considered in this paper.

To check for the effect on the stability boundary of the
choice of flow regime transition criteria, three different laws
for the Fanning factor have been used. In particular, the laws
adopted are:

1. the Blasius law

\[ f'(Re) = \frac{0.079}{Re^{0.25}} \]  (28)

2. a law with smooth transition between laminar and
turbulent flow

\[ f'(Re) = \sqrt{f_{lam}^2 + f_{turb}^2} \]  (29)
with

\[ f_{lam} = \frac{16}{Re} \]
\[ f_{turb} = \frac{0.079}{Re^{0.25}} \]  (30)

3. the formulation by Churchill [12], with an \( \varepsilon/D \) ratio of
0.0001

The different relationships are reported in Figure 2.

The linear stability analysis of the system in Eqs. (27) for
the positive flow steady-state conditions, identified by the
relationship

\[ \frac{2 Gr}{\pi^2} \cos \gamma = \frac{L}{D} f'(Re^{Re}) Re^{Re^2} \]  (31)

can be easily performed by the usual linearization

\[ \tau = \frac{t}{T} = \frac{p D L}{2 \mu} \]
\[ F_\sigma = \frac{\alpha T}{(L / 2)} \]  (25)

Concerning the boundary conditions it is assumed that
the heat flux is uniform and constant in both the heating and
the cooling sections. With these conditions, the problem has
two steady-state solutions, one for positive and the other for

Figure 2 - Different friction laws adopted for the toroidal loop
techniques. In this work, stability maps have been obtained mapping the real part of the eigenvalue of the linearized ODE system having maximum norm, $z_\tau$. This quantity measures the degree of amplification or damping of perturbations and is used to define the conditions for neutral stability ($z_\tau = 0$) and the regions of stability ($z_\tau < 0$) or instability ($z_\tau > 0$).

The maps in Figures 3 to 5 clearly show the impact of the choice of the friction law on the linear stability boundary. In this connection it must be remarked that the use of a simple power law for the Fanning factor, which is the usual choice in literature on thermosyphon loops, gives a very poor representation of the situation occurring when a realistic law is adopted.

3. METHOD OF ANALYSIS

The technique here adopted for analyzing the linear stability of natural circulation loops starts with the discretization of the governing equations by a suitable numerical method. Though applications reported in the present paper will mostly refer to the upwind explicit numerical method (see e.g., Mincowycz, 1988), chosen for its relevance for some thermal-hydraulic system codes, the proposed method of analysis is quite general and has been applied to different first order and second order numerical methods, both explicit and implicit [2].

Time and space discretization of the governing PDEs by the finite difference numerical method results in a system of algebraic equations having the form

$$ F(\mathbf{y}_n, \mathbf{\delta y}_n; \Delta \tau, \Delta s, \text{physical parameters}) = 0 \quad (32) $$

which relates the values of the vector of unknowns, $\mathbf{y}_n$, at the times $\tau^n$ and $\tau^{n+1}$, grid parameters and physical parameters. To get information about stability of steady-state conditions as predicted by the numerical method, this equation can be linearized by perturbing the vector $\mathbf{y}_n$ around steady-state conditions

$$ y_\tau^n = y_\tau + (\delta y)_n^n \quad y_\tau^{n+1} = y_\tau + (\delta y)_n^{n+1} \quad (33) $$

where $y_\tau$ represents the steady-state value of $y_\tau$ and is a solution of the equation:

$$ F(\mathbf{y}_n, \mathbf{\delta y}_n; \Delta \tau, \Delta s, \text{physical parameters}) = 0 \quad (34) $$

As in classical stability analyses, a linearized relationship relating perturbations at the two different time levels is reached

$$ (\delta y)_n^{n+1} = - (J_{y_\tau} y_\tau^{n+1})^{-1} \cdot J_{y_\tau} y_\tau^n \cdot (\delta y)_n^n \quad (35) $$

where the Jacobian matrices of $F$ with respect to $y_\tau^n$ and $y_\tau^{n+1}$ calculated at steady-state conditions appear.

It is clear from the above that assuming an exponential amplification or damping of perturbations

$$ (\delta y)_n^{n+1} = e^{\Delta \tau \cdot (\delta y)_n^n} \quad (36) $$

stability can be discussed considering the eigenvalues of the matrix

$$ A = - (J_{y_\tau} y_\tau^{n+1})^{-1} \cdot J_{y_\tau} y_\tau^n \quad (37) $$

where the inverse of the jacobian with respect to $y_\tau^{n+1}$ always exist for any meaningful numerical method. Considering the spectral radius of $A$, $\rho(A)$, the quantities

$$ \Delta \rho = \rho(A) - 1 \quad \text{and} \quad \tau_\tau = \max \left( \frac{\ln(\rho(A))}{\Delta \tau} \right) \quad (38) $$

are then defined, which are negative for stable conditions and positive for unstable ones. They can be used as a measure of the margin in excess to neutral stability and are related to the predicted degree of amplification or damping of perturbations. The latter with respect to $\Delta \rho$ has the advantage that its range does not depend directly on the time step adopted in the numerical solution and can be compared with the eigenvalues of ODE systems representing the considered physical problem obtained by other techniques (see Sect. 2).

Both quantities are anyway useful to set up quantitative stability maps, showing the location of the stability boundary.
and the different degrees of amplification or damping of the numerical solution. Examples of such maps are provided in this paper.

A characteristic which makes the present methodology useful in numerical study of fluid dynamic instabilities is that the effect of truncation error on linear stability can be quantitatively assessed for different discretizations and numerical methods, thus providing a guidance for choosing the optimum nodding and time step in transient analyses.

On the other hand, the unstable conditions identified by the methodology may have both physical and numerical origin; therefore, care must be taken in discriminating between them in the obtained linear stability maps.

4. RESULTS

The governing equations for Welander’s problem have been discretized adopting four different numerical schemes and stability was studied by the methodology discussed in Sect. 3. In particular, the following schemes have been considered (see e.g., [13]):

- the first order upwind explicit scheme (Forward Time Upwind Space = FTUS);
- the first order upwind implicit scheme (Implicit Time Upwind Space = ITUS);
- the second order upwind explicit scheme (Warming-Beam);
- the second order upwind explicit scheme (Warming-Beam).

As an example, the discretized forms of the energy and momentum equations adopted for the first order upwind explicit scheme are the following:

- \( q \geq 0 \)
  \[
  q_{n+1} = q_n + \Delta \tau \sum_{i=1}^{N-1} \frac{q_n + q_{n+1}}{2} - \varepsilon (q_n^H)^\gamma \Delta \tau 
  \] (41)

where
  \[
  C = \frac{q_n^H \Delta \tau}{\Delta s} \quad \Delta s = \frac{1}{N-1} \] (42)

On the other hand, second order accurate methods provide a better performance in predicting stability than first order ones. As it can be noted in Figure 6, even with only ten nodes and with a ten times lower time-step than those in Figure 8, they are anyway able to approximate with relatively high fidelity the actual stability boundary. These results suggest that second order terms appearing in truncation error of first order methods have the greatest impact on numerical diffusion. This conclusion has been independently confirmed by the stability analysis of the mentioned modal solution in which a second order term equivalent to the one brought about by discretization through the FTUS method was purposely added [2].

For the rectangular loop, theoretical stability maps have been presented by Vijayan and Austregesilo [6] characterized by a bell shaped neutral stability line in the plane \( Gr_{m-stm} \). Similar stability maps were obtained by linearization of a first order upwind explicit numerical method applied to the energy equation with the usual Euler explicit integration scheme for the momentum equation. The values of the dimensionless parameters adopted in obtaining the maps are the following:

- \( D/L = 9.25 \times 10^{-4} \), \( \Delta \tau = 10^{-3} \)
- \( p = 22.26 \), \( b = 0.6744 \) (43)

The results are reported in Figure 7 in the form of quantitative stability maps based on \( \Delta \rho \). Various numbers of nodes have been used in the legs and in the heater and cooler to point out the effect of spatial discretization on the shape and the extension of the unstable area.

It can be noted that with only 40 nodes in the whole loop (10 in each leg and 10 in both heater and cooler) prediction of instability conditions is very poor compared with the solutions obtained with greater number of nodes. In fact, increasing the number of nodes in the legs, larger and larger unstable areas are found in the considered parameter space.

The effect of discretization in the heater and the cooler appears less relevant than that in the legs as it can be noted comparing the maps with 50 nodes in the legs and 10 and 30 nodes, respectively, in both heater and cooler. This result might be explained arguing that the pipes connecting the heater and the cooler have a dominant role in determining the timing for amplification of thermal perturbations which lead to instability and the greatest care should be put in their discretization.

Also in the case of the toroidal loop a first order upwind explicit method has been used to set up stability maps. In this case, \( z \) was preferred to allow for direct comparison with the maps obtained by the modal solution. Figure 8 reports the obtained maps showing the modifications which the stability boundary undergoes due to numerical diffusion.
FTUS with 50 nodes and $\Delta \tau = 10^{-3}$

ITUS with 50 nodes and $\Delta \tau = 10^{-3}$

Warming-Beam with 10 nodes and $\Delta \tau = 10^{-4}$

Figure 6 - Welander's Problem: stability maps for first order and second order numerical schemes ($\nu = 1.75$)

Figure 7 - Rectangular loop: quantitative stability maps obtained with a first order upwind explicit method

(D/L = 9.25$x10^{-4}$, $\Delta \tau = 10^{-3}$, $p = 22.26$, $b = 0.6744$, $V/V_{heater} = 8.1$, $\phi = 3$)
Figure 8 - Toroidal loop: quantitative stability maps obtained with a first order upwind explicit method
(Churchill law, $\varepsilon/D = 0.0001$, L/D = 100, $\Delta \tau = 10^{-7}$)

5. CONCLUSIONS

In the present work, stability maps were set up for three different single-phase thermosyphon loops making use of finite difference numerical schemes. Though the presented sample applications are related to systems which may be considered idealized from many respects, it is clearly understood that more complex systems could be addressed with a reasonable effort.

In selecting the configuration of the three loops attention was paid in choosing boundary conditions representative of the whole range of systems considered in literature. So, temperature controlled heat transfer, heat flux controlled heat transfer and mixed boundary conditions were assumed.

In the cases of Welander's problem and of the rectangular loop with mixed boundary conditions, the form of dimensionless governing equation was taken from previous literature. For the toroidal loop, it was preferred to propose dimensionless equations including the Reynolds number as flow variable. This form, which is fairly general and could have been used even in the case of the other thermosyphon loops, has the advantage that transition between laminar and turbulent flow is easily obtained, making use of a traditional formulation for friction factors.

The obtained results clearly show the effect of truncation error on stability prediction: greater stability than in reality is generally predicted when a coarse nodalization is used. As observed for Welander's problem, 2nd order methods provide a reasonable representation of the conditions for stability with a much smaller number of nodes than 1st order ones. Moreover, explicit numerical methods must be also preferred since they show smaller diffusion than implicit ones.

It must be remarked that convergence of the obtained numerical stability maps to the theoretical ones is always possible increasing the spatial discretization detail or using higher order methods [2]. In this respect, it can be noted that the adopted methodology allows a direct control of truncation error effects and may be used as a guidance for judging reliability of nonlinear transient calculations performed making use of the same numerical method.

Finally, it must be observed that linear stability analysis is only the first step in studying stability conditions in single-phase loops. In practical applications, nonlinearities play an important role in defining the actual conditions for stability, since finite perturbations may lead the system into instability, though it is linearly stable, due to the presence of subcritical bifurcations. A certain degree of spreading in stability boundaries has been in fact observed during experiments [14] and can be experienced also by nonlinear calculations close to the linear stability boundary, as a consequence of the differences in initial conditions. Some degree of conservatism must be therefore considered in making use of linear stability analysis results.
NOMENCLATURE

Roman Letters

A  Area \[ \text{m}^2 \]
A  Matrix
a, b  Coefficients of the friction law
C  Courant number
\( C_p \)  Specific heat at constant pressure \[ \text{J/(kg K)} \]
D  Diameter \[ \text{m} \]
F  Vector equation expressing a numerical scheme
\( F_{nod(q)} \)  Source-sink heat transfer multiplier
f'  Fanning factor
g  Gravity \[ \text{m}^2/\text{s} \]
Gr  Grashof number
h  Heat transfer coefficient \[ \text{W/(m}^2\text{K)} \]
J  Jacobian matrix
k  Heat Conduction \[ \text{W/(m K)} \]
L  Loop length
N  Number of nodes
\( \mathcal{N} \)  Order of the algebraic system
P  Power \[ \text{W} \]
Pr  Prandtl number
p  Fanning factor coefficient
Q  Volumetric flow rate \[ \text{m}^3/\text{s} \]
q  Dimensionless volumetric flow rate
q'  Heat flux \[ \text{W/(m}^2\text{K)} \]
R  Friction parameter (from Welander) \[1/\text{s} \]
Re  Reynolds number based on pipe diameter \[ - \]
S  Axial coordinate along the loop \[ \text{m} \]
St  Stanton number
T  Fluid temperature \[ \text{K} \]
t  Time \[ \text{s} \]
U_ij  Heat conductance in the cooler
V  Loop volume \[ \text{m}^3 \]
W  Mass flow rate \[ \text{kg/s} \]
\( y \)  General vector of unknowns
\( \Xi \)  Perimeter \[ \text{m} \]
\( \rho \)  Fluid density \[ \text{kg/m}^3 \]
\( \rho(A) \)  Spectral radius
\( t \)  Dimensionless time \[ \text{s} \]
\( \phi \)  Ratio of loop volume on leg volume
\( \Psi_1 \)  Coefficient of sinus 1st term in modal expansion
\( \Omega_1 \)  Coefficient of cosinus 1st term in modal expansion
\( \omega \)  Ratio of actual to steady-state flow rate

Subscripts

cooler  cooler
heater  heater
lam  laminar
leg  leg
m  modified
max  maximum value
r  real
ss  steady-state
turb  turbulent
w  wall
Wel  Welander
\( y_0, y_{n+1} \)  related to the corresponding vectors
0  reference or initial value

Superscripts

n  Value at time level n
n+1  Value at time level n+1
s  Steady-state value
*  Dimensionless value

Abbreviations

FTUS  Forward Time Upwind Space
ITUS  Implicit Time Upwind Space
ODE  Ordinary Differential Equation
PDE  Partial Differential Equation
SB  Stability Boundary

REFERENCES


