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OF SINGLE-PHASE NATURAL CIRCULATION STABILITY

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ABSTRACT

The paper discusses the effects of numerical discretisation in the prediction of single-phase natural circulation as observed in a simple experimental thermosyphon loop. Stability of steady-state conditions is particularly addressed, making use of a calculation model purposely set up for studying natural circulation in loops having general shape and distribution of heat sources and sinks. The model is equipped with both a classical upwind explicit first order numerical scheme and a low diffusion scheme devised on the basis of the Warming-Beam second order upwind explicit method.

The results obtained by the model making use of different degree of detail in spatial discretisation, different time steps and numerical methods are compared with the predictions of a thermal-hydraulic system code making use of similar nodalisations. The main emphasis of the discussion concerns the effects of first order numerical discretisation as pointed out by both calculation models, which make the prediction of the dynamic behaviour of the considered loop a rather challenging problem. The need and the feasibility of adopting higher order numerical methods for studying unsteady single-phase natural circulation is one of the outcomes of the analysis.

INTRODUCTION

The prediction of natural circulation instabilities observed in single-phase thermosyphon loops involves unexpected difficulties due to both their chaotic behaviour and the need to adopt modelling techniques capable to reproduce the observed complexity without introducing spurious numerical effects.

Single-phase natural circulation loops are, in fact, classical examples of systems exhibiting deterministic chaos [1], as even the simplest configurations may show non-periodic flow oscillations characterised by a remarkable sensitivity to initial conditions. This feature results in an essential unpredictability of transient behaviour, which frustrates any attempt to exactly reproduce a specific observed evolution in time, though the mechanistic laws governing the phenomenon are known and a reasonably accurate knowledge of initial and boundary conditions is available.

Welander proposed a meaningful physical explanation for the occurrence of such instabilities in an early paper [2] and many other works have been performed later on to study the behaviour of different loop configurations both from the experimental and the theoretical points of view (see e.g., Refs. [3] to [11]). In particular, single-phase thermosyphon loops may be brought into instability by particular combinations of buoyancy and friction forces, which determine self-sustaining oscillations in flow rate leading ultimately to alternate flow reversals. This is due to the existence of at least two admissible fixed points (i.e., steady-state conditions), which are characterised by forward and reverse flow respectively. When neither the positive nor the negative fixed points are stable, the system oscillates with growing amplitude around one of them, until it becomes attracted by the other, thus repeating the process with inverted flow.

Much of the chaotic nature of the system is due to this mechanism of successive flow reversals. In fact, at each oscillation flow reversal may or may not occur depending on the value of flow rate obtained during the swing; if the amplitude of the deviation from steady-state conditions is not enough to cause inverted flow, another swing must take place before reversal. This determines a situation in which system trajectories starting from very close initial conditions may considerably depart from each other in subsequent time as a result of different behaviour at flow reversals.

To further enhance the resulting sensitivity to initial conditions, the usual bifurcating behaviour observed in many nonlinear systems is also present. Especially in cases in which physical parameters are such that the system is close to marginal stability, the extent of the initial perturbation may be decisive in determining subsequent stable or unstable behaviour.

Given the mentioned fundamental unpredictability of chaotic systems, there is no real hope, nor interest, in exactly reproducing the time history of any measured variable by numerical models. In similarity with the case of fluid turbulence, the main aim of modelling an unstable loop flow is to reasonably predict the observed qualitative and quantitative behaviour in terms of average amplitude and frequency of oscillations, together with growing and damping factors. With respect to the mere prediction of stable steady-state natural circulation, which pose anyway serious problems in the definition of friction and heat transfer laws appropriate for the specific addressed heating and flow conditions (which do not match with the usual forced flow formulations), the simulation of dynamic loop behaviour reveals very sensitive to apparently minor details. Even though a reasonable accuracy is achieved in predicting steady-state flow rate or temperature distribution, stability may be anyway poorly represented owing to inaccuracies having physical or numerical origin.
The latter category of inaccuracies is particularly relevant when using space and time discretised approximations of the partial differential equations governing the fluid-dynamic and thermal loop behaviour. This constitutes an almost mandatory choice while dealing with real systems, characterised by the presence of complicated features, like variable cross section or distributed heat transfer with loop walls. In fact, simple models resulting in low order ordinary differential equation systems can be developed only making use of simplifying assumptions which result too restrictive for practical purposes.

Previous work [12-14] has shown that the application of first order finite difference discretisation techniques to the one-dimensional energy balance equation may reveal remarkably inefficient, as the related truncation error often results in strong damping of system oscillations, thus biasing the calculated behaviour towards stability. In particular, it was demonstrated [12] that dissipative effects introduced by the appearance of second order derivative term in the modified equation of usual first order upwind numerical methods (both explicit and implicit) are mainly responsible for the observed damping of oscillatory behaviour. The physical counterpart of this numerical effect is the introduction of axial heat conduction within the fluid, which changes the dynamics of transport of temperature information along the loop with respect to the case of pure advection.

As a consequence, second order numerical methods behave naturally better as they do not involve any such spurious conductive effect. In particular, it was shown [12] that converged stability maps can be obtained making use of second order numerical methods with rather coarse nodalisations, while first order methods always show a variable degree of diffusion even with large numbers of nodes, also depending on the adopted time-step and their explicit or implicit nature. This diffusion constitutes a major problem in stability analyses and must be as far as possible ruled out.

In the present paper, an experimental single-phase natural circulation loop is considered as reference physical system to perform sensitivity analyses on flow stability by numerical models. In particular, a numerical model set up with the purpose of simulating the dynamic behaviour of single-phase loops characterised by general shape and distribution of heat sources and sinks in dimensional form is used. The program [15] is equipped with two different upwind explicit numerical methods: a classical first order method and a low diffusion method based on the Warming-Beam second order scheme (see e.g., [16]). The thermal-hydraulic system code RELAP5/MOD3.2 [17] is also adopted in the analysis for purpose of comparison with the predictions of the developed numerical model.

As in the mentioned previous works [12-14] numerical effects were studied with reference to idealised flow loops whose governing equations were expressed in dimensionless form, the main emphasis in the paper is in showing how these effects may be also of importance in the case of a real life experimental loop.

REFERENCE EXPERIMENTAL FLOW LOOP

The loop considered in this paper is named MTT-1 and is installed at the Dipartimento di Termoenergetica e Condizionamento Ambientale (DITEC) of Genova University [11]. It mainly consists of two copper horizontal tubes and two Plexiglas vertical tubes connected with steel 90° bends (Figure 1).

The loop is 1245 cm high and 1480 cm wide. The lower heating section consists of an electrical heating wire wound on the outer surface of the copper tube, heated with a power up to 3200 W. The upper heat sink is made of a coaxial cylindrical heat exchanger fed with tap water flowing in the secondary side annulus. High values of water flow rate are adopted for the secondary flow in order to achieve a rather uniform temperature all along the annulus. An expansion tank open to the atmosphere is provided, in order to compensate for temperature variations occurring in the fluid due to transient heat transfer. Calibrated thermocouples are installed in the loop in order to measure temperatures in relevant locations within the fluid.

Figure 1 - Considered experimental flow loop (from Ref. [11])

Many experimental tests have been carried out using the loop in order to study the relation between the power provided to the system and the stability of the flow [11]. However, it was not possible to define a threshold value of the power separating stable and unstable behaviour. As in other experiments [9], the obtained results show that the stability of the loop depends on many factors such as the geometry, the friction forces and the intensity of the buoyancy forces.

In particular, the stability behaviour of the loop has been analysed by varying heating power from 300 W to 3200 W; all the obtained data show an unstable flow behaviour. Flow reversals occur in different ways at the different power levels; at sufficiently high power the classical behaviour characterised by periods of growing amplitude oscillations separated by flow reversals are observed (see e.g., Figure 2). The loop shows some degree of asymmetry, exhibiting more oscillations when the flow is clockwise than when it is counter-clockwise. The increase in power has the effect to shorten the waiting period between flow reversals; moreover, it results in larger temperature difference spikes across the heater, which are observed when the flow rate vanishes before reversal.
DESCRIPTION OF THE NUMERICAL MODEL

The model adopted in the present analysis [15] has been set up for dealing with general single-phase thermosyphon loops. Nevertheless, in the choice of the reference loop description and of the related models it was tried to adopt the simplest solutions compatible with a realistic representation of experimental loops. The main goal of this choice is to obtain a simple but reliable tool for both linear and non-linear stability analysis of natural circulation in single-phase apparatuses.

In this aim, the loop is subdivided into segments characterised by constant diameter, flow area and heat structure parameters. Each segment is in turn subdivided into an arbitrary number of fluid nodes facing a corresponding heat structure node. At the moment, only circular cross sections and annular structures are considered, but with minor changes to the input processing section, the program could easily deal with any other suitable geometry.

The environment outside the flow loop is treated parametrically, defining appropriate values for fluid temperature and heat transfer coefficient to be applied to the external surface of each node heat structure. In this way, both heat losses and heat transfer to a secondary fluid can be dealt with. Even the modelling limitation consequent to the assumption of constant outer fluid temperature and heat transfer coefficient can be easily removed.

The governing equations are written making use of the Boussinesq approximation; therefore, while the energy equation is solved by both space and time discretisation, resulting in a system of algebraic equations for nodal temperatures, a single ordinary differential equation is obtained for momentum along the whole loop. Heat structures are also considered: the related energy balance equation is solved in a lumped parameter form, taking into account conduction effects in the structure thickness.

The resulting governing equations are:

- fluid energy balance in the i-th node
  \[ \frac{dW_i}{dt} = \frac{\rho_f \beta_f}{1} \sum_{i=1}^{N} \Delta s_i g_i T_{f,i} - F(W) \frac{dW}{W} \]
  (3)
  where
  \[ 1 = \sum_{i=1}^{N} \left( \frac{\Delta s_i}{A_{f,i}} \right) \]
  \[ F(W) = \sum_{i=1}^{N} \left[ \frac{\Pi_{f,i}}{2 \rho_f A_{f,i}} f'(R_e_i) + \sum_{i=1}^{N} \frac{K_i}{2 \rho_f A_{f,i}} \right] \]

- energy balance in heat structures
  \[ \frac{dT_{s,i}}{dt} = -\lambda_{w,i} T_{s,i} + Q_{w,i} \]
  (5)
  where
  \[ \lambda_{w,i} = \lambda_{w,i}^{in} + \lambda_{w,i}^{out} \]
  \[ Q_{w,i} = \frac{dW_i}{\rho_f C_{pw,i} + \lambda_{w,i}^{out} T_{f,i}^{in} + \lambda_{w,i}^{out} T_{f,i}^{out}} \]
  \[ \lambda_{w,i}^{in} = \frac{\hat{h}_{w,i}^{in} \Pi_{w,i}}{A_{w,i} \rho_f C_{pw,i}} \]
  \[ \lambda_{w,i}^{out} = \frac{\hat{h}_{w,i}^{out} \Pi_{w,i}}{A_{w,i} \rho_f C_{pw,i}} \]

The above equations are then discretised in space and time:

- discretised fluid energy balance
  \[ T_{f,i}^{n+1} = T_{f,i}^{n} + \frac{W^n}{\Delta s_i} \left( f_i(T_{f,i}^{n-1} - T_{f,i}^{n-1}) + S_i^{n+1} \Delta t \right) \quad (i = 1, \ldots, N) \]
  (8)

- momentum balance
  \[ W^{n+1} = \frac{W^n + \rho_f \beta_f \Delta s_i}{1 + \frac{F(W^n)}{W^n} \frac{\Delta s_i}{1 + \frac{dW}{W}}} \]
  (9)

- energy balance in the structures
  \[ T_{s,i}^{n+1} = T_{s,i}^{n} e^{-\lambda_{w,i}^{out} \Delta t} + \frac{Q_{w,i}^{in}}{\lambda_{w,i}^{out}} \left[ 1 - e^{-\lambda_{w,i}^{out} \Delta t} \right] \]
  (10)

In the fluid energy balance equation (8), the symbols \( T_{f,i}^{n} \) and \( S_i^{n+1} \) represent the "donored" temperatures at junctions separating the fluid nodes and the source term. Both variables are evaluated according to the following numerical discretisation schemes:

- a simple first order upwind scheme, which is similar to the semi-implicit algorithm adopted by RELAP5 when applied to single-phase flow;
- a low diffusive scheme, obtained on the basis of the semi-implicit algorithm adopted by RELAP5 when applied to single-phase flow;

Both the schemes, which can be optionally selected by the user, share the characteristic to be explicit and to use upwind differences. These are both very useful features, as they result in simplicity in solving the corresponding difference equations and in a very desirable transportive property of the discretised energy balance, which determines advection of temperature information in the appropriate direction. On the other hand, explicitness in time discretisation brings about the usual limitations on the maximum allowed Courant number.

The relevant difference between the two schemes lies in the order of accuracy. The explicit upwind scheme is first order accurate, while the Warming-Beam scheme is second order
accurate. As already mentioned, this makes a remarkable difference when dealing with natural circulation loop stability, as first order schemes involve spurious dissipation having the capability to dramatically change the predicted amplification of flow and temperature perturbations.

Constitutive relationships are adopted for evaluating heat and momentum transfer between the fluid and the heat structures. In particular, the Churchill relationships [18] are adopted for friction factors in the laminar and turbulent regimes and the Colburn correlation [19] is used for heat transfer coefficients.

Steady-state versions of the described equations are iteratively solved for purpose of initialisation, whenever it is required to start the calculation close to a stable or unstable fixed point. This is particularly useful when the above transient relationships are linearised by perturbation around the fixed point; the evaluation of the eigenvalues of the linearised equation system provides information about linear stability and allows setting up stability maps. Two different programs have been therefore developed for performing transient and linear stability analyses, named TRANLOOP and LINELOOP respectively. Only the former has been used to produce the results described in the present paper.

FLOW LOOP DESCRIPTION

In the aim to compare the results provided by RELAP5 and TRANLOOP, similar nodalisations have been adopted for the two programs. The one adopted by RELAP5 is described in detail in a companion paper [20]. In the same reference, it is shown that RELAP5 reasonably predicts the main features of the experiment, including the periods of oscillations and flow reversals. Only minor changes (related to user options) have been introduced with respect to the original input deck, in order to equalise calculation assumptions in the two codes. In the base nodalisation adopted for TRANLOOP (Figure 3), the same number of nodes has been adopted for the various parts of the flow loop as in RELAP5 model, but the surge line and the expansion tank (see Figure 1) was not modelled. This simplification is possible as the presence of the expansion tank in the RELAP5 nodalisation has the purpose to avoid pressure excursions during loop transients; this problem has no relevance in the case of an incompressible fluid as assumed in TRANLOOP.

A time step of 0.1 s is used in the long term in both codes. This results in a relatively small Courant number (in the order of 0.1 at steady-state flow rate) which enhances diffusion in first order numerical schemes.

RESULTS

Figures 4 to 9 summarise the relevant results obtained by the analysis of discretisation effects on predicted stability. The results have been obtained for a reference case with a heater power of 1800 W. In order to highlight numerical effects only, in TRANLOOP calculations the heat transfer coefficient on the outer cooler surface has been assigned to the value calculated by RELAP5 in the corresponding case.

In particular, Figures 4 and 5 show that in the base calculation cases TRANLOOP and RELAP5 provide a very similar representation of the dynamic behaviour of the system. The results also compare qualitatively well with the results reported in Figure 2 for temperature difference across the heater.

The extent at which these results appear to be affected by numerical diffusion can be estimated considering Figures 6 and 7, where flow rate trends obtained using nodalisations involving (nearly) doubled or halved numbers of nodes are shown. Doubling the number of nodes does not change the qualitative response of the codes, though some more dynamic (i.e., less damped) behaviour can be observed. On the other hand, halving the number of nodes strongly enhances numerical diffusion, leading to the prediction of stable steady-state conditions, in contrast with experimental observation. A striking similarity between the behaviour of the two programs can be noted in this respect, except for the occasional convergence to the forward or the reverse flow steady-state conditions, which depends on the differences in the initial transients and also provides and example of sensitivity to initial conditions.

A straightforward way of reducing numerical diffusion in a first order upwind numerical scheme is to change continuously the time step in order to obtain a Courant number, \( C = \frac{w|\Delta t|}{\Delta s} \), as far as possible close to unity. Of course, to be applicable this remedy requires also nearly uniform spatial discretisation and fluid velocity, as it is in the present case. As \( C = 1 \) is also the numerical stability limit for the scheme, care must be taken to keep a reasonable departure from unity to avoid meaningless results. Figure 8 reports the results obtained by TRANLOOP with the base nodalisation imposing \( C = 0.95 \); in comparison with the previous trends obtained with a fixed \( \Delta t = 0.1 \text{ s} \), a greater frequency of flow reversals is noted, showing that the predicted behaviour is affected by a lower degree of damping. The obtained trend is also very similar to the one provided by the use of the low diffusion numerical scheme, shown in Figure 9; in fact, an increase in the frequency of flow reversals is noted even in this case, testifying for a comparable reduction in numerical diffusion.

CONCLUSIONS

The obtained results show that most of the conclusions obtained in previous work [12-14] in relation to the effect of numerical diffusion in idealised natural circulation loops are applicable also in the case of a real experimental apparatus.
Figure 4 - Flow rate and heater temperature difference from TRANLOOP 1st order scheme with the base nodalisation.

Figure 5 - Flow rate and heater temperature difference from RELAP5 with the base nodalisation.

Figure 6 - Flow rate from TRANLOOP 1st order scheme with nearly doubled and nearly halved numbers of nodes.

Figure 7 - Flow rate from RELAP5 with nearly doubled and nearly halved numbers of nodes.

Figure 8 - Flow rate and heater temperature difference from TRANLOOP 1st order scheme with the base nodalisation and an imposed Courant number of 0.95.

Figure 9 - Flow rate and heater temperature difference from TRANLOOP low diffusion scheme with the base nodalisation.
In particular, notwithstanding the greater complexity introduced by the presence of heat structures, of secondary flow loops and of realistic closure relationships for heat and momentum transfer at the walls, numerical diffusion plays a relevant role in determining the predicted dynamic behaviour of the system.

The use of numerical schemes characterised by a higher order of accuracy with respect to the ones presently adopted in system codes appears the only feasible and effective way to solve the problem in general cases. This provides further motivation to develop and apply low diffusion numerical techniques as the one included in the model adopted in the present work.

NOMENCLATURE

Roman Letters

- A: flow area \([m^2]\)
- C: Courant number \(= n \Delta t / \Delta s\)
- \(C_p\): specific heat \([J/(kgK)]\)
- \(f'\): Fanning friction factor
- g: gravity \([m/s^2]\)
- \(h\): overall wall-fluid heat transfer conductance \([W/(m^2K)]\)
- K: singular pressure drop coefficient
- N: number of nodes
- \(q''\): volumetric power \([W/m^3]\)
- Re: Reynolds number \(= \mu A / W D\)
- s: axial coordinate \([m]\)
- S: source term \([K/s]\)
- t: time \([s]\)
- T: temperature \([K]\)
- w: velocity \([m/s]\)
- W: mass flow rate \([kg/s]\)

Greek Letters

- \(\beta\): isobaric expansion coefficient \([K^{-1}]\)
- \(\lambda\): reciprocal time constant \([s^{-1}]\)
- \(\Pi\): perimeter \([m]\)
- \(\rho\): density \([kg/m^3]\)

Subscripts

- f: fluid
- i: node index
- k: index of singular pressure drop
- w: wall
- wf: wall to fluid

Superscripts

- frict: friction
- in: inner
- out: outer
- n, n+1: time levels

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