Linear and Nonlinear Analysis of Density Wave Instability Phenomena

Ambrosini, W.; Di Marco, P. And Ferreri, J.C.
LINEAR AND NONLINEAR ANALYSIS OF DENSITY WAVE INSTABILITY PHENOMENA

Ambrosini, W.1; Di Marco, P1. and Ferreri, J.C.2

1 Università degli Studi di Pisa, Italia
2 Nuclear Regulatory Authority, Argentina

ABSTRACT

In this paper the mechanism of density-wave oscillations in a boiling channel with uniform and constant heat flux is analysed by linear and nonlinear analytical tools. A model developed on the basis of a semi-implicit numerical discretization of governing partial differential equations is used to provide information on the transient distribution of relevant variables along the channel during instabilities. Furthermore, a lumped parameter model and a distributed parameter model developed in previous activities are adopted for independent confirmation of the observed trends. The obtained results are finally put in relation with the picture of the phenomenon proposed in classical descriptions.

1. INTRODUCTION

Stability of vapour generation in a boiling channel is an important operating and safety concern in various industrial applications, including steam generators and boiling water nuclear reactor cores [1] [2]. Coupling of boiling channel dynamics with external feedback mechanisms, e.g., due to recirculation loop and, in the latter case, to neutron kinetics effects, makes the related instability phenomena rather complex, showing an interesting variety of parametrical trends which justifies detailed analyses of the overall plant behaviour. Besides, the upcoming need to install boiling systems on spacecrafts raises the issue of determining the stability of boiling channels in the absence of buoyancy.

The different mechanisms leading to boiling channel instabilities have been recognised and classified in basic works on the subject, which also provided explanations for their occurrence based on engineering understanding of the related phenomena [3-7]. Attention is focused here on density wave instabilities, which are considered to be at the basis of the most frequently observed instability occurrences in boiling channels. In particular, in reactor cores density waves are believed to trigger the observed coupled oscillations of channel flow rate and neutron flux.

The classical descriptions of the density wave instabilities are generally related to a boiling channel with an imposed constant pressure drop across it. In this case, the single-phase and the two-phase region pressure drops oscillate in counterphase as a consequence of waves of heavier or lighter fluid travelling from the inlet to the outlet channel section. In a simple channel with no distributed friction, inlet and outlet pressure drop perturbations should be almost in phase with the local fluid density perturbation, being positive or negative when lumps of denser or lighter fluid respectively are passing. For some values of system parameters, the delays involved in the propagation of these density waves (hence the name) are such as to make the steam generation process unstable, resulting in the observed self-sustained oscillations of flow rate. A relevant consequence of this postulated behaviour is that the period of density-wave oscillations should be around the double of the fluid transit time, in order to make a complete cycle of positive and negative flow perturbations in the time needed to propagate the corresponding heavier and lighter density waves from the inlet to the outlet of the boiling channel (see e.g., [5-7]).

This picture of the phenomena has been criticised by Rizwan-uddin some years ago [8] on the basis of a nonlinear model adopting homogeneous equilibrium balance equations for simulating the behaviour of a boiling channel. In particular, considering a specific test case at relatively high inlet subcooling, it was shown that the pressure drop at the exit of the channel does not always increase simultaneously with increasing the exit fluid density, thus showing that there may be no direct correlation between the oscillations in pressure drop and the waves of denser and lighter fluid propagating along the channel. Moreover, it was noted that in such a case the period of oscillations is considerably larger than twice the transit time and that during an oscillation the mixture density seems to increase and decrease almost simultaneously in the whole two-phase region. It was then
recalled that also some experimental data by Saha, Ishii and Zubér [9] show a longer period of oscillation compared to twice the transit time. Finally, it was concluded that, in the considered region of the parameter space, the phenomenon seems to be more likely governed by mixture volumetric flux rather than by mixture density oscillations.

It is interesting to note that descriptions proposing density-wave oscillations as a phenomenon mainly governed by flow perturbations also appeared in literature (see e.g. [10]). In addition, previous studies [11] identified various types of both Ledinegg and density-wave instabilities, the latter characterised by different characteristic oscillation periods. This clearly suggests a more complex picture of these phenomena than in simplified explanations of the basic mechanism.

The above summarised criticisms by Rizwan-uddin are the starting point for the present paper, having the objective to provide additional information for discussing the mechanisms at the basis of density wave oscillations. Both the linear stability conditions and the transient distribution of relevant thermal-hydraulic variables during oscillations are here considered, in the aim to discuss the pictures previously proposed for describing the density-wave oscillation mechanism.

In particular, use is made of linear and nonlinear predictive tools developed in previous activities [12-15] and of a discretised model recently set up on the basis of a semi-implicit numerical method. The latter model has been developed as an application to boiling channel stability of a methodology of analysis adopted in previous works for assessing the effect of numerical methods on stability predictions in single-phase thermosyphon loops [16-17].

As in the paper which stimulated the present analysis [8], the homogeneous equilibrium balance equations are used for providing the basic mathematical description of boiling channel dynamics. This is a clear limitation in the present work as well as in the work by Rizwan-uddin, which must be duly considered to avoid extending the validity of the obtained results beyond the reasonable limits of a discussion around the very basic mechanism of density-wave instabilities. However, though it is obviously understood that such a simplified description will miss some details of the actual system dynamics, it is nevertheless believed that the homogeneous equilibrium model can provide useful information for understanding the gross behaviour of a boiling channel under unstable conditions, without the complications coming from the use of more sophisticated models.

To partly compensate for this limitation, the results of calculations previously performed with a two-fluid system code [18] are also taken into consideration for supporting the obtained findings.

2. DESCRIPTIONS OF DENSITY-WAVE INSTABILITIES

Let us now start the discussion, presenting two representative descriptions of density-wave phenomena appeared in previous literature. The first one, by Kakac and Liu [7], presents a classical point of view, in which the phenomenon is governed by lighter and heavier fluid pockets propagating along the channel. The second one, by Podowski [10], proposes a more complex mechanism in which both flow and density perturbations play a key role.

Kakac and Liu [7] describe density-wave instabilities with reference to an horizontal evaporator duct followed by an exit restriction. Inlet and outlet plenum pressures are kept constant and the vapour generation is assumed constant as well. A sudden outlet pressure drop perturbation, e.g. resulting from a local microscopic increase in void fraction, is assumed to trigger instability by propagating at the speed of sound a corresponding low pressure pulse to the channel inlet.

As pressure in the inlet plenum is assumed to be constant, this causes an increase in inlet flow. An enthalpy wave is therefore generated which propagates towards the boiling boundary, where it is transformed into a denser fluid wave reaching the exit restriction after some additional time. The total time needed to the inlet flow perturbation to affect the outlet section is the particle transit time through the channel.

In turn, the denser fluid reaching the outlet section produces a pressure drop perturbation of opposite sign with respect to the one which started the process. Therefore, it is expected to experience an inlet flow decrease and the propagation of a lighter fluid wave toward the exit section, thus closing the cycle. As the overall process requires the propagation of a positive and a negative wave to be completed, the period for density-wave oscillations should be in the range of twice the transit time. Since in less idealised systems distributed friction occurs, this relation is only approximate.

On the other hand, Podowski [10] starts considering the different speeds of propagation of velocity perturbations in the single-phase and the two-phase regions. It is observed that in the liquid region these perturbations travel at the speed of sound, covering the non-boiling length in a very short time; when appearing at the boiling boundary they give rise to increases or decreases in the local void fraction which propagate at the kinematic velocity which characterises density-waves, closer to the vapour velocity than to the speed of sound. The changes in both flow and void fraction result in pressure drop perturbations, with a different effect of velocity variations from the hydrodynamic and the thermal points of view; in fact, decreasing the flow would tend to decrease also the pressure drop, but this effect is counteracted by the corresponding increase in void fraction due to greater vaporisation.

Since the density-waves propagate slowly in the two-phase region, the overall constraint of a constant pressure drop across the channel may lead to conditions in which the single-phase region and the two-phase region pressure drop oscillate in counterphase and the oscillations are self-sustained.

It is clear that Rizwan-uddin’s criticisms mainly apply to descriptions of the first kind, while they are in not in disagreement with the second one. Summarising, the main concerns raised in his paper [8] are the following:

- the exit pressure drop does not always increase or decrease simultaneously with exit density;
- the period of density wave oscillations may be considerably longer than twice the fluid transit time;
• as a whole, the phenomenon is more likely governed by flow oscillations than by density waves.

In the following, it will be tried to provide material for discussing the applicability of these criticisms.

3. DISCRETISED BOILING CHANNEL MODEL

The reference physical model consists of a boiling channel coupled with constant pressure boundary conditions at the inlet and the outlet sections (the so-called “parallel channel” conditions), with both singular and distributed pressure drops and an arbitrary distribution of heat flux from the heating wall. The singular pressure drops are assumed to be concentrated at the inlet and at the outlet of the channel. Neither heater dynamics nor neutron feedback are here considered for the sake of simplicity and in order to focus the attention on purely thermal-hydraulic phenomena.

The homogeneous equilibrium model (HEM) is adopted for describing the boiling channel flow behaviour. The related mixture balance equations are:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho j}{\partial z} = 0 \]

\[ \frac{\partial \rho j}{\partial t} + \frac{\partial \rho j^2}{\partial z} + \frac{\partial p}{\partial z} = -\rho_0 \left[ \frac{f}{D_t} + 2K_\delta \delta_j(z) + 2K_\delta \delta_j(z-L) \right] \frac{\rho j^2}{2} \]  

(1)

where \( \delta_j \) is a dimensional Dirac's delta function [m\(^{-1}\)]. These equations were chosen to be essentially the same as those adopted in previous works [12-15], in order to keep consistency with the results there obtained. The only difference is the introduction of the heat flux distribution function, \( f_j \), which has been used in subsequent work to perform sensitivity analyses on the effect of non-uniform channel heating. However, in this work a uniform channel heating is always assumed, i.e. \( f_j = 1 \).

It must be emphasised that the adopted form of the thermal energy balance is a very convenient approximation of the exact equation, obtained by neglecting the energy dissipation due to friction and the contribution to the enthalpy change due to fluid expansion along the channel. The former approximation is rather obvious for most of the systems of practical interest; on the contrary, the latter one, though fully justified in the operating conditions characterising boiling water reactors, may be relatively inaccurate in very extreme conditions, not considered in the present work, characterised by both low pressure and low channel power.

The above equations are then made dimensionless to increase generality of the calculated results. In this aim, the following definitions are adopted:

\[ \rho^* = \frac{\rho}{\rho_j}, \quad h^* = \frac{h-h_j}{h_{ts}-h_j}, \quad j^* = \frac{j}{w_0}, \quad t^* = \frac{t w_0}{L} \]

\[ \zeta^* = \frac{z}{L}, \quad p^* = \frac{p}{\rho_j w_0^2}, \quad G^* = \rho^* j^* \quad Fr = \frac{w_0^2}{g L} \]

\[ \Lambda = \frac{f L}{2D_h N_{pch}} \quad N_{pch} = \frac{q_s \Pi L}{\rho_j w_0 h_{ts} A f}, \quad N_{sub} = \frac{h_j-h_{ts} v_{ts}}{h_{ts} v_{fs}} \]  

(2)

where an arbitrary reference pressure is assumed to evaluate the fluid properties which are considered independent from the local value of pressure. From this assumption it follows

\[ \rho^* = \frac{1}{\rho_j (v_j + x_\rho v_p)} = \frac{1}{1+h^*} \quad \text{if} \quad h^* \geq 0 \]

\[ \rho^* = 1 \quad \text{if} \quad h^* < 0 \]  

(3)

which are the dimensionless state relationships for our problem. In the calculations an appropriate smoothing is adopted for density when \( h^* \) approaches 0, in order to avoid numerical problems arising from the discontinuity in derivatives. The velocity scale appearing in equations (2), \( w_0 \), is here defined to be the inlet velocity. As a consequence of these assumptions, in steady-state conditions it is \( j^*_i = 1 \) and it is also \( G^* = \rho^*_i \) everywhere in the channel.

The resulting dimensionless equations are:

\[ \frac{\partial \rho^*}{\partial t} + \frac{\partial \rho^* j^*}{\partial z} = 0 \]

\[ \frac{\partial G^*}{\partial t} + \frac{\partial (j^* G^*)}{\partial z} + \frac{\partial p^*}{\partial z} = \frac{-\rho_j}{Fr} \left[ (\Lambda + K_\delta \delta_j(z') + K_\delta \delta_j(z'-L) ) G^* \right] \frac{G^*}{\rho} \]

\[ \frac{\partial \rho^* h^*}{\partial t} + \frac{\partial (j^* h^* G^*)}{\partial z} = N_{pch} f_j(z') \]

(4)

where \( \delta_j \) and \( f_j \) are respectively a dimensionless Dirac's delta function and the heat flux distribution function, both depending on \( z' \). In the case of forward flow at both the inlet and the outlet sections, constant values of dimensionless pressure in the two plena and \( h^*_m = -N_{sub} \) are assumed as boundary conditions for transient analysis.

Solution of the above equations is obtained by a semi-implicit numerical technique [19]. The boiling channel is subdivided into \( N_e \) equally spaced nodes (see Figure 1) in which dimensionless pressure, density and enthalpy are defined. At the junctions separating adjoining volumes dimensionless mass fluxes are defined, resulting in a classical staggered mesh scheme.

Mass and energy balance equations are discretised in conservative form, giving:

\[ \rho_{i+1}^* = \rho_i^* + \frac{G_{i+1}^* - G_{i}^*}{\Delta z} \Delta t^* \quad (i = 1, \ldots, N_e) \]

(5)

![Figure 1 - Boiling channel discretisation](image-url)
\[
\left( \rho_i^* h_i^* \right)^{t+1} = \left( \rho_i^* h_i^* \right)^{t} + \left[ G_{i+1}^{*} - G_{i-1}^{*} \right] \left[ \begin{array}{c} \Delta \rho_i^* \\ \Delta h_i^* \end{array} \right] + N_{i-1} \delta_f(z_i^*) \Delta t
\]

where \( \delta_f(z_i^*) \) is an appropriate average over the i-th node of the heat flux distribution factor and the quantities \( h_i^* \) and \( \rho_i^* \) are the "donated" enthalpies to be evaluated according to the sign of the dimensionless mass flux at each junction, in compliance with the rules of upwind differencing.

The evaluation of \( G_{i+1}^{*} \) and \( G_{i-1}^{*} \), whose values are required in the above equations, is performed using the momentum equation, discretised in space and time in the form:

\[
G_{i+1}^{*} = \frac{\rho_i^{*2}}{\rho} \left[ \frac{G_{i+1}^{*2}}{\rho} \right] - \frac{1}{2} \left( \rho_i^{*2} + \rho_i^{*1} \right) \Delta t \Delta y_n^* - \left[ \Lambda + K_i \delta(z_i^*) + K_i \delta(z_i^* - 1) \right] \frac{G_{i+1}^{*1} \Delta t}{G_{i+1}^{*1} \Delta y_n^*} + \left( \rho_i^{*1} - \rho_i^{*2} \right) \Delta t \]

(6)

where \( \delta \) is a Dirac's delta function operating over a discretised domain and \( \overline{G}_{i+1}^{*} \) and \( \overline{G}_{i}^{*} \) are angle average mass fluxes. This equation is firstly put in the form:

\[
G_{i+1}^{*1} = C_{G_i} \left( p_i^{*1} - p_i^{*2} \right) + O_i^* \quad (i = 1, \ldots, N_n + 1)
\]

(8)

in which \( C_{G_i} \) and \( O_i^* \) are appropriate constants and the \( \delta \) of dimensionless mass at the inlet and the outlet are used for \( i = 1 \) and \( i = N_n + 1 \). Then, mass and energy balance equations are discretised in non-conservative form and combined in order to give a relationship between junction mass fluxes:

\[
\Gamma_{i+1}^{*1} G_{i+1}^{*1} = \Gamma_{i}^{*1} G_{i}^{*1} + \Theta_i^* \quad (i = 1, \ldots, N_n)
\]

(9)

Combining the last two equations results in a tridiagonal algebraic system in nodal pressures:

\[
- G_{i+1}^{*1} C_{G_i} P_i^{*1} + \left[ G_{i+1}^{*1} C_{G_i} + G_{i+1}^{*1} C_{G_i} \right] P_i^{*1} - G_{i+1}^{*1} C_{G_i} P_i^{*1} = 0 \quad (i = 1, \ldots, N_n)
\]

(10)

which is solved by a standard algorithm giving the nodal dimensionless pressures. Equation (8) is then used to update junction mass fluxes, involved in the above described conservative forms of mass and energy balance equations, to evaluate \( \rho_i^{*1} \) and \( h_i^{*1} \). A common problem at this stage is related to mass error, that is the difference between the nodal density calculated by mass conservation and the one corresponding to the value given by state relationships. In the present case, to avoid accumulation of mass error, the value of nodal dimensionless density at the new time-step is assumed equal to the one corresponding to the new dimensionless enthalpy. This is justified by the fact that the mass error at each time step is very small.

Calculation initialisation is performed computing steady-state conditions at constant dimensionless inlet mass flux. Once the channel pressure drop corresponding to the prescribed channel parameters \( (N_{sub} \cdot N_{pch}, \Lambda, K_i, K_z, f_i^*(z_i^-)) \) has been evaluated in steady-state, this is imposed as boundary condition to the boiling channel during the transient analysis, which is initiated by an impulse perturbation in outlet plenum pressure.

In order to get a more clear picture of stability conditions in the parameter space, the above described discretised equations are also linearised by perturbation, according to a methodology successfully adopted in previous works for analysing single-phase thermosyphon loop stability [16-17]. The discretised equations and the related boundary conditions are firstly written in the form of a system of \( 3N_n + 1 \) nonlinear relationships

\[
F(y^*, y^{*1}, \Delta y, \Delta y^{*1}, N_{sub}, N_{out}, \ldots) = 0
\]

(11)

involving the values of the components of the vector of independent variables

\[
y^* = \left[ p_i^*, p_{i+1}^*, \ldots, p_{i+N_n}^*, h_i^*, h_{i+1}^*, \ldots, h_{i+N_n}^*, G_i^*, G_{i+1}^*, \ldots, G_{N_n+1}^* \right]
\]

(12)

at two subsequent time-steps \( t = t^* \) and \( t = t^{*1} \). Then, small oscillations around the steady-state conditions are assumed, by writing

\[
y^* = y^* + (\delta y)^* \quad y^{*1} = y^* + (\delta y)^{*1}
\]

(13)

Finally, by introducing these definitions in the discretised equations (11) and considering that in steady-state it must be

\[
F(y^*, y^{*1}, \Delta y, \Delta y^{*1}, N_{sub}, N_{out}, \ldots) = 0
\]

(14)

second order terms are neglected and the linear relationship between old and new time-step perturbations around the steady-state conditions is found to have the form

\[
(\delta y)^{*1} = -J_y^{*1} \cdot (\delta y)^* - J_{y^{*1}} \cdot (\delta y)^* = A \cdot (\delta y)^*
\]

(15)

where \( J_y^* \) and \( J_{y^{*1}} \) are the Jacobian matrices of the nonlinear discretised equations with respect to \( y^* \) and \( y^{*1} \) evaluated at steady-state conditions:

\[
J_y^* = \left. \frac{\partial F}{\partial y} \right| \quad J_{y^{*1}} = \left. \frac{\partial F}{\partial y^{*1}} \right|
\]

(16)

Therefore, calculation of the spectral radius \( \rho(A) \) of matrix \( A \) allows discussing stability; in particular, \( \rho(A) \) greater than unity defines unstable conditions. Furthermore, putting:

\[
\rho(A) = |\lambda_{max}|
\]

(17)

where for the eigenvalue having maximum modulus, \( \lambda_{max} \), the following expansion is adopted

\[
\lambda_{max} = \exp \left( \frac{\ln(\rho(A))}{\Delta y} \right) = e^{-\Delta y \cdot \frac{1}{\Delta y} \cdot \cos(\Delta y) + \sin(\Delta y) \cdot \Delta t}
\]

(18)

it follows that

\[
z_{\Delta} = \frac{\ln(\rho(A))}{\Delta y} \quad \text{and} \quad z_{\Delta} = \frac{1}{\Delta y} \arccos \left[ \frac{\text{Re}(\lambda_{max})}{\rho(A)} \right]
\]

(19)

The first quantity measures the degree of damping (if negative) or amplification (if positive) of small perturbations and can be used to set up stability maps; the second one allows to evaluate the dimensionless frequency of the fastest growing (or less damped) perturbation, given by:

\[
f_{\Delta} = \frac{z_{\Delta}}{2\pi}
\]

(20)

It will be interesting in the following sections to compare the resulting dimensionless period of small oscillations, \( T_{\Delta} = f_{\Delta}^{-1} \), with the dimensionless transit time of the fluid mixture in the channel, defined as:

\[
T_{\Delta} = \sum_{i=1}^{N_n} \frac{\Delta t_i}{G_i}
\]

(21)
Considering the above described methodology for linear stability analysis, it can be noted that it is conceptually similar to the usual stability analysis of ordinary differential equation systems. Nevertheless, it differs from it because it is applied to both space and time discretised algebraic equations.

In addition to provide information on the stability of the addressed physical system when well converged solutions are considered, this methodology is capable to highlight the effects of numerical grid parameters (node size and time-step duration) on stability predictions (see e.g., [16-17; 21]). For the results presented in this work a reasonable degree of numerical convergence was ascertained; however, in presenting stability maps the adopted grid parameters will be reported to precisely identify the calculation assumptions.

### 4. DISTRIBUTED PARAMETER AND LUMPED PARAMETER MODELS

A distributed model and a lumped parameter model have been developed in previous activities [12] to investigate boiling channel stability by linear and nonlinear techniques. In both cases, the adopted HEM balance equations are essentially the same as the ones used in the above described discretised model. As these models have been already described elsewhere [12-15], only the main modelling choices adopted in their development will be here summarised.

In the distributed parameter model (Figure 2), suitable for the linear stability analysis of single-channel and multi-channel configurations, the HEM partial differential equations are firstly perturbed and the steady-state solution is eliminated to give two linear equation sets (one for the single-phase region and the other for the two-phase region) which are then Laplace transformed. After conversion from the time to the frequency domain, integration of the obtained ordinary differential equations along the channel axis allows to reach closed forms for the Laplace transform of the perturbed variables. Combination of these expressions and conversion to dimensionless form allows expressing the transforms of the dimensionless pressure drop perturbations in the single-phase and the two-phase regions as a function of the inlet dimensionless volumetric flux perturbation

\[
\delta(\Delta \rho_{\sigma}) (s) = \tilde{\Gamma}_{ij}(s) \delta \tilde{\rho}_{\sigma}(s) \quad (22)
\]

\[
\delta(\Delta \rho_{\sigma}^*) (s') = \tilde{\Pi}_{ij}^*(s') \delta \tilde{\rho}_{\sigma}^*(s') \quad (23)
\]

where \( s' \) is the dimensionless complex variable. In the particular case of a single channel with constant inlet and outlet plenum pressures (i.e., in the “parallel channel” condition), the constraint of zero total pressure drop perturbation allows to combine the single-phase and the two-phase pressure drop transfer functions to reach a characteristic equation (see Figure 2)

\[
1 + \tilde{\Pi}_{ij}^*(s')/\tilde{\Gamma}_{ij}(s') = 0 \quad (24)
\]

whose roots can be discussed to identify the neutral stability conditions.

The lumped parameter model is based on the same HEM balance equations adopted for the distributed parameter model. However, in this case the boiling channel is subdivided into three moving boundary nodes, two of them representing the single-phase region, the other one adopted for the two-phase region (Figure 3). Balance equations are integrated over each channel node and converted into dimensionless form to obtain a set of nonlinear ODEs. Semi-empirical relations are needed to this aim. A heater model is also available, which anyway is not adopted in the present analysis. Linearisation by perturbation of the governing equations and application of the Routh-Hurwitz criterion allows studying the linear stability conditions. Conversely, numerical integration in time of the ODEs by the Adams predictor-corrector method makes it possible to study the nonlinear transient behaviour of the boiling channel.

### 5. MODEL APPLICATION

#### 5.1 Linear Stability

The models described in the previous sections have been applied to relevant parameter cases. For purpose of comparison with the lumped and the distributed parameter models, a stability map has been set up with the linearised discretised model for a nearly horizontal channel without distributed friction and uniform heat flux. In the aim to obtain a reasonable convergence, 48 nodes have been used in the channel and a maximum Courant number of 0.9 has been adopted to minimise numerical diffusion. The map has been obtained calculating the values of \( z_k \) (see Eq. (15)) within the range 0-30 for \( N_{mph} \) and 0-20 for \( N_{mub} \) with a step of 0.25 for both coordinates. Contour plots providing a clear picture of stability conditions have been built up on this basis.

Such a plot is shown in Figure 4, in which curves at constant \( z_k \) are shown. The typical features of the density wave and Ledinegg instability regions clearly appear. Comparing with Figure 5, in which the marginal stability boundaries are shown for the distributed and the lumped parameter models, a good agreement is observed.
Some difference can be anyway noted which is partly due to the different methods adopted to set up the maps; in particular, it clearly appears that the construction of a contour plot from discrete values of $z_R$ makes it impossible to carefully describe the thin Ledinegg instability region at low inlet subcooling. On the other hand, the good agreement of the marginal stability boundary for density wave oscillations at high inlet subcooling predicted by the models with the simplified Ishii criterion (see e.g., [22]) can be observed. In addition to provide a reliable description of the marginal stability boundary, the discretised model allows an interesting quantitative look to the degree of damping or amplification throughout the stable and the unstable regions. In particular, it is shown that on either side of the stability boundary $z_R$ changes smoothly except in the vicinity of the zero exit quality line ($N_{puch} = N_{puch}$), where a sort of “continental platform” has its leading edge. The sharpness of this edge is certainly decreased with respect to the actual trend, owing to the use of smoothed fluid properties across the boiling boundary.

The ratio between the fluid transit time and the period of oscillation at the onset of instability as obtained by the discretised model throughout the Ishii-Zuber plane is shown in Figure 6. It can be noted that the period of density wave oscillations is not predicted to be everywhere equal to twice the transit time. In particular, a continuous distribution of frequencies exists going from low inlet subcooling to the boundary for Ledinegg instability, where excursive behaviour occurs, characterised by an infinite period of oscillation.

Figure 7 shows similar results obtained for a vertical channel with distributed friction. The comparison with Figure 6 indicates that the main factor affecting the distribution of the frequency of oscillation in the Ishii-Zuber plane is the location of the Ledinegg instability region, limiting the zone in which the period of oscillation may have a finite value.

5.2 Nonlinear behaviour

Figures 8 and 9 show stable limit cycles obtained by the discretised model and the lumped parameter model in the case of very large Froude number (e.g. horizontal channel, or microgravity conditions) with friction losses concentrated at the inlet and at the outlet of the channel. The similarity of the trends obtained by the two models can be noted.

It is also interesting to investigate by the discretised model the distribution of relevant variables along the channel during these limit cycle oscillations. The results obtained for density, pressure, mass flux and momentum flux are shown in Figure 10. Some relevant features appear in the figure:

- all the variables oscillate with a relatively long frequency due to the vicinity of the operating point to the Ledinegg instability region (see Figure 6);
- density oscillations occur almost simultaneously along the whole two-phase region as the time needed to propagate density waves is much shorter than the period of oscillations;
- singular pressure drops over the inlet and the outlet sections oscillate almost counterphase; it is interesting to note that, due to the absence of distributed losses and gravity along the channel, very small pressure drops...
occur in the non-boiling region, while in the boiling region acceleration losses appear;

mass flux perturbations propagate instantaneously in the non-boiling region and with some delay in the boiling one; on the other hand, momentum flux perturbations oscillate nearly counterphase in the non-boiling and the boiling region, as a consequence of the constraint on the overall pressure drop;

- oscillations in outlet pressure drop do not occur in phase with those in outlet density and outlet mass flux; however, they are obviously in phase with oscillations in outlet momentum flux.

Figure 8 – Limit cycle obtained by the discretised model

Figure 9 – Limit cycle obtained by the lumped parameter model

Figure 10 – Distribution of relevant variables during a limit cycle as predicted by the discretised model

\( N_s = 48, \ C_{\text{max}} = 0.9, \ Fr = 10^5, \Lambda = 0.0, K_{\text{m}} = 6, K_{\text{e}} = 2, N_{\text{pch}} = 14.5, N_{\text{sub}} = 9, f' = 1 \)
As this quantitative information is related to a particular region of the Ishii-Zuber plane, where the period of oscillations is much larger than the fluid transit time, a similar analysis has been repeated at a lower value of inlet subcooling near the “nose” of the stability boundary, where the period of oscillations is closer to twice the transit time.

Figure 11a shows in the form of a contour plot the results obtained for density along the channel during slowly diverging oscillations in such a case. It can be seen that, owing to the reduced length of the liquid region, the delays in propagation of density perturbations are enhanced. In this case the oscillations in the outlet pressure drop (Figure 11b) are more closely related with those in outlet channel density and in exit mass flux (Figure 11c). This behaviour suggests a combined role of both velocity perturbations and density waves in this parameter range.

Results similar to the ones previously described were found in the case of vertical channels with distributed friction. This suggests to consider density-wave oscillations as the result of both velocity and density perturbations, whose combination occurs in different ways in the various regions of the parameter space.

In order to better support the previous findings at least in the case of low inlet subcooling, Figure 12 shows the results obtained in a previous work [18] by the RELAP5/MOD3 code for density wave instability in a downscaled boiling water reactor simulator with nearly saturated water at channel inlet. The calculation assumed that neutron kinetics could be simulated in the experimental apparatus by an appropriate processing of core level measurements, to account for density-reactivity feedback. Though this situation is more complex than the single channel one addressed in the above analyses, since it includes also loop and point kinetics effects, it is clear from the figure that density does not change simultaneously along the channel; in particular, density-wave propagation is clearly predicted by the two-fluid model, in similarity with the results obtained by the homogeneous equilibrium model at low inlet subcooling (see Figure 11a).

6. CONCLUSIONS
This study aimed to contribute to the understanding of the mechanisms characterising density-wave oscillations in boiling channels under different operating conditions. The
starting point of the analysis has been the criticism raised by Rizwan-uddin [8] in a previous work against the classical description of the phenomenon.

The use of different linear and non-linear analysis tools and, in particular, of a recently developed space and time discretised model allowed to get quantitative information about the behaviour of a boiling channel under constant pressure drop conditions.

On the basis of the obtained data, it is possible to conclude that:

- considering channel conditions close to the stability boundary at high inlet subcooling, it was possible to confirm the picture proposed by Rizwan-uddin [8] for boiling channel oscillations, in which:
  - the period of oscillations is considerably longer than twice the fluid transit time;
  - the outlet density does not oscillate in phase with outlet pressure drop;
  - the density-wave phenomenon looks mostly governed by flow perturbations rather than by actual density wave propagation;

- on the other hand, considering slowly diverging oscillations at low inlet subcooling it was found that the system behaves in greater similarity with what suggested by classical descriptions:
  - the period of oscillations is closer to twice the fluid transit time;
  - outlet density oscillations are more closely correlated with outlet pressure drop oscillations;
  - density wave propagation can be observed along the channel;

- even in the case of high Froude numbers with friction concentrated at the inlet and the outlet sections, the period of density-wave oscillation is roughly equal to twice the channel transit time only in a limited range of parameters; therefore, the deviation of the ratio $T_{\text{osc}} / T_{\text{fast}}$ from 0.5 cannot be explained with the effects of distributed friction or gravity;

- the continuous distribution of the ratio $T_{\text{osc}} / T_{\text{fast}}$, which has been found throughout the Ishii-Zuber plane shows that the Ledinegg instability can be found in the limit of density wave instabilities at zero oscillation frequency; as a consequence, the position of the Ledinegg stability boundary, depending on the static characteristics of the channel, strongly affects the distribution of $T_{\text{osc}} / T_{\text{fast}}$.

In summary, as it was partly expected, the obtained results show that both flow perturbations and density waves superimpose and interact during boiling channel oscillations, resulting in the observed complex behaviour. As a consequence, comprehensive descriptions of the phenomenon should account for the combination of both, avoiding to focus on a single effect.

The relative weight of either kind of mechanism is different in different regions of the parameter space, showing that at low inlet subcooling the classical descriptions of density wave phenomena may be possibly considered applicable, while at high inlet subcooling they clearly are inadequate to explain the observed behaviour.

NOMENCLATURE

Roman Letters

- $A$: cross section area [m$^2$]
- $A$: matrix
- $C_{\text{max}}$: maximum Courant number in the channel
- $C_p$: constant
- $D_h$: hydraulic diameter [m]
- $F$: vector function of the discretised equations
- $Fr$: Froude number [-]
- $f$: friction factor [-] or frequency [s$^{-1}$]
- $f_q$: heat flux distribution function [-]
- $G$: mass flux [kg/(m$^2$s)]
- $g$: gravity [m/s$^2$]
- $h$: specific enthalpy [J/kg]
- $J$: Jacobean matrix
- $j$: volumetric flux [m/s]
- $K$: singular pressure drop coefficient [-]
- $L$: channel length [m]
- $N_p$: number of nodes [-]
- $N_{\text{pch}}$: phase change number (see Eqs.2) [-]
- $N_{\text{sub}}$: subcooling number (see Eqs.2) [-]
- $O$: constant
- $p$: pressure [Pa]
- $q_0'$: heat flux [W/m$^2$]
- $T$: period [s]
- $t$: time [s]
- $v$: specific volume [m$^3$/kg]
- $w_0$: reference velocity scale [m/s]
- $y$: vector of independent variables
- $z$: axial coordinate [m]
- $z_h$, $z_i$: real and complex exponents

Greek Letters

- $\Gamma$: constant or transfer function
- $\delta$: Dirac’s delta function [-]
- $\Theta$: constant
- $\Lambda$: dimensionless friction parameter [-]
- $\lambda$: eigenvalue [-]
- $\Pi_h$: heated perimeter [m]
- $\Pi$: transfer function
- $\rho$: density [kg/m$^3$] or spectral radius

Subscripts

- $d$: dimensional value
- $ex$: exit
- $f$: liquid
- $fast$: fastest growing perturbation
g vapour
h heated
i node index
in inlet
j volumetric flux
max maximum
trans transit
y\* related to y

Superscripts

n referred to the old time-step
n+1 referred to the new time-step
a dimensionless quantity
\bar{a} node average quantity
\hat{a} donated quantity
\tilde{a} Laplace transformed variable

REFERENCES